1. Prove that if a heuristic is consistent, it must be admissible.  
   Construct an admissible heuristic that is not consistent. HINT: Use  
   proof by induction

Answer:

N’ = future state

N = current state

C = cost to go from n to n’ with action a

For consistency

A heuristic function is consistent if every n every successor n’ of n generated by action a

h(n) <= c(n,a,n’) + h(n’)

for admissible :

If h(n’) were to overestimate the true cost, then we'd have:

h(n’) > c(n, a, n’) + h(n)

But since h(n) is admissible (it doesn't overestimate), we have:

h(n) ≤ c(n, a, n’) + h(n)

Combining these inequalities, we get:

h(n’) ≥ h(n) ≥ c(n, a, n’) + h(n)

This implies that h(n’) is admissible, as it doesn't overestimate the true cost to reach the goal from n’.

Constructing an admissible heuristic that’s not consistent

To construct an admissible heuristic that is not consistent, you can use proof by induction. Here's a simple example:

Consider a state space with two states: A and B, with a goal state G.

* The true costs are as follows:
  1. •C(A, G) = 10 (cost from A to G)
  2. •C(B, G) = 5 (cost from B to G)

Now, let's define a heuristic function h(n) for this space:

* Base Case:
  1. h(A) = 0 (heuristic for the goal state is 0)
  2. h(B) = 0 (heuristic for the goal state is 0)
* Inductive Step:
  1. For any node n in the state space, we define its heuristic h(n) as follows:
     1. •If n ≠ G (i.e., n is not the goal state), then h(n) = C(n, G) + 1.
     2. •If n = G (i.e., n is the goal state), then h(G) = 0.

Explanation:

* The heuristic for the goal state G is 0, which is admissible.
* For any other state n, the heuristic value h(n) is set to the true cost from n to the goal G plus 1. This makes sure that h(n) is always an overestimate of the true cost (because we add 1 to the actual cost), making it admissible.
* However, this heuristic is not consistent because it doesn't satisfy the consistency property for all states and their successors. Specifically, for state B and its successor G, we have: h(B) = C(B, G) + 1 = 5 + 1 = 6
* C(B, G) = 5 . This violates the consistency condition, as h(B) > C(B, G) + h(G).

So, this example demonstrates that you can construct an admissible heuristic that is not consistent by carefully selecting heuristic values for different states.